

Calculations of the cross sections for the neutron acceleration, slowing down, and capture by the isomer ^{180m}Hf

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The cross sections for inelastic acceleration, slowing down, and capture are calculated for the isomer ^{180m}Hf in the energy range of the incoming neutron from 1 keV to 5 MeV. Below 0.7 MeV the energy transferred to the neutron in the collision is positive on the average, i.e., the isomer is acting as a "neutron accelerator".

In the neutron collisions with nuclei which are in a long-lived excited (isomeric) state, inelastic neutron acceleration (INNA) is possible, as a result of which the emitted neutron carries away the excitation energy¹ (or some part of it). Recently this reaction has been observed experimentally with thermal neutrons for the two isotopes ^{152m}Eu , Ref. 2, and ^{180m}Hf , Ref. 3; here in the last case the cross section for acceleration of thermal neutrons was 52 ± 13 b. Owing to the multiple acceleration of neutrons an inversely populated medium can in principle act as a neutron accelerator.⁴ This allows consideration of the possibility of increase of the multiplication coefficient of a system containing fissile isotopes with a fission threshold.⁵ In order to calculate the neutron spectra in real systems one must know the magnitude and energy dependence of the neutron cross sections for inelastic acceleration, slowing down, and capture by specific isomers. In addition, the same data are needed for taking into account the INNA reaction in calculations of element formation in stars.⁶ In the present paper these cross sections are calculated for ^{180m}Hf in the energy interval 0.001-5 MeV with a statistical model and the method of coupled channels using the technique described in Refs. 7 and 8.

In Fig. 1 the adopted level scheme of ^{180m}Hf is shown.⁹ The use of the statistical model for the calculation of neutron cross sections for ^{180m}Hf , for which the K -forbiddenness for electromagnetic transitions increases the lifetime by 16 orders of magnitude (in comparison with the single-particle estimate), is justified by the fact that for neutron reactions via the compound-nucleus there are reasons to assume that this forbiddenness does not exist.¹⁰⁻¹³ In particular, the probabilistic theoretical estimate of the cross section for the acceleration of thermal neutrons in the statistical model¹⁴ for this isomer agrees satisfactorily with experiment.³

The calculations of the cross sections were performed with the Hauser-Feshbach-Moldauer formulas. The initial data for these calculations are sticking coefficients (T -coefficients) which for deformed nuclei are calculated with the method of coupled channels.

The choice of the parameters of the optical potential is made in the spirit of the SPRT method of Lagrange,¹⁵ i.e., starting from the requirement of simultaneous description of the experimental strength functions S_0 and S_1 , the range of potential scattering R' , and the energy dependence of the total cross section $\sigma_t(E)$. In this approach the limits of vari-

ation of the T -coefficients are rather narrow and determine the admissible set of parameters of the nuclear potential. For example, for isomers with a spherical shape this approach permits description of the cross section for the INNA reaction with accuracy¹⁶ ~ 20%. The neutron cross sections for

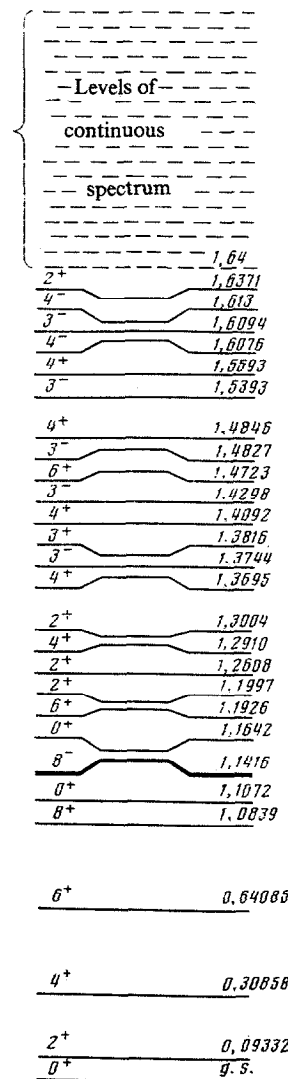


FIG. 1. The adopted scheme of the levels of ^{180m}Hf .

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TABLE

Level
 J_π
 A, E, I
 σ, b

E, M
0.001
0.00E
0.01
0.1
0.05
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.5
1.75
2.0
2.5
3.0
4.0
5.0

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the deformed fissile nuclei ^{235}U and ^{239}Pu agree with experiment with the same or better accuracy.⁸ Final calculations are performed for the potential of a deformed nucleus in the form

$$V(r, \vartheta) = V_0 f(x_1) + 4iW_D \frac{d}{dx_2} f(x_2) + V_{s_0} (\hat{\mathbf{i}}\hat{\sigma}) \frac{1}{ar} \frac{d}{dx_1} f(x_1), \quad (1)$$

$$x_1 = (r - R_i)/a_i, \quad R_i = r_{0i} A^{1/3} [1 + \beta_2 Y_{20}(\vartheta)],$$

$$f(x_i) = (1 + e^{x_i})^{-1}$$

with the following values of the parameters:

$$V_0 = -47.0 + 0.3 \cdot E, \text{ MeV},$$

$$W_D = \begin{cases} -5.03\sqrt{E} & \text{for } E < 1 \text{ MeV} \\ -4 - 1.03\sqrt{E} & \text{for } E \geq 1 \text{ MeV} \end{cases}$$

$$V_{s_0} = -6.5 \text{ MeV}, \quad \beta_2 = 0.35, \quad r_{01} = r_{02} = 1.25 \text{ F},$$

$$a_1 = 0.65 \text{ F}, \quad a_2 = 0.47 \text{ F}.$$

Here E is the incident-neutron energy in MeV, $\hat{\mathbf{i}}$ is the orbital-angular-momentum operator, and $\hat{\sigma}$ are the Pauli matrices.

This set of parameters gives $S_0 = 1.1 \cdot 10^{-4}$, $S_1 = 1.7 \cdot 10^{-4}$, $R' = 7.3 \text{ F}$, and $\sigma_t(1 \text{ keV}) = 21.2 \text{ b}$, which is in agreement with the experimental data ($S_0 = (0.7 \pm 0.5) \cdot 10^{-4}$ for ^{180}Hf and $S_0 = (1.8 \pm 0.2) \cdot 10^{-4}$ for ^{181}Ta ; $R' = 7.3 \pm 0.3 \text{ F}$ for ^{180}W and $\sigma_t(1 \text{ keV}) = 20 \pm 5 \text{ b}$ for natural hafnium¹⁷). The value of S_1 for the isotope ^{180}Hf is not known.

We note that the authors of Ref. 18 for description of

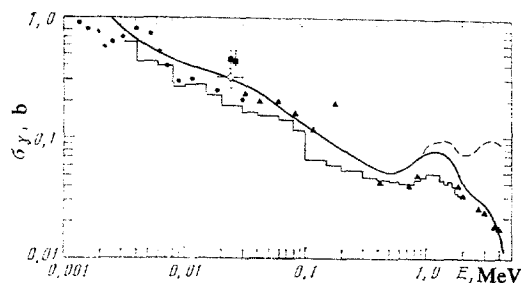


FIG. 2. Comparison of the calculated cross section for radiative capture with experiment.^{18,23} The solid curve corresponds to $\sigma_{n\gamma}$ and the dashed one to $\sigma_{n\gamma} + \sigma_{n\gamma n}$.

the cross section for capture by ^{180}Hf use the value $S_0 = 1.84 \cdot 10^{-4}$ measured for ^{177}Hf . Here a significantly smaller value of S_1 than the one given above was obtained. However, there are no reasons to assume that the values of S_0 for the isotopes ^{177}Hf and ^{180}Hf are equal. A significant change of S_1 will result in a poorer description of the energy dependence of the total cross section and the cross section for radiative capture in the energy range of order of hundreds of keV, where the contributions from s and p waves are similar. The set of parameters (1) gives $\sigma_t(0.5 \text{ MeV}) = 7.2 \text{ b}$ and the estimate for natural hafnium $\sigma_t(0.5 \text{ MeV}) = 7.5 \text{ b}$ (spread of the data from 6 to 8 b). The cross section $\sigma_\gamma(E)$ in the interval 0.1–1 MeV, where the p wave makes a large contribution, is described with accuracy 30% (see Fig. 2), which also justifies the use of the adopted value of S_1 .

The main uncertainty for incident-neutron energies

TABLE I. Calculated neutron cross sections for ^{180m}Hf .

Level I_i^π	3	4	5	7	9	19	Contin- uous spec- trum	The ($n, \gamma n$) reaction	Pure capture (without ($n, \gamma n$))	
$\Delta E_i, \text{ MeV}$	0.833	0.501	0.058	0	-0.051	-0.331				
$\sigma, \text{ b}$	$\sigma_{m \rightarrow 3}$	$\sigma_{m \rightarrow 4}$	$\sigma_{m \rightarrow 5}$	$\sigma_{m \rightarrow 7}$	$\sigma_{m \rightarrow 9}$	$\sigma_{m \rightarrow 19}$	σ_{cont}	$\sigma_{n, \gamma n}$	$\sigma_{n, \gamma}$	σ_t
$E, \text{ MeV}$										
0.0011	0.47	2.83	2.34	3.31	-	-	-	0.03	4.73	21.2
0.005	0.20	1.24	0.97	2.30	-	-	-	0.02	1.85	13.1
0.01	0.14	0.88	0.68	1.82	-	-	-	0.01	1.22	11.1
0.05	0.07	0.45	0.37	0.88	-	-	-	-	0.51	8.3
0.1	0.05	0.32	0.33	0.56	0.02	-	-	-	0.34	7.3
0.2	0.06	0.35	0.42	0.82	0.07	-	-	0.01	0.32	7.5
0.3	0.07	0.42	0.50	0.79	0.09	-	-	0.01	0.30	7.4
0.4	0.08	0.32	0.51	0.91	0.11	-	-	0.01	0.29	7.3
0.5	0.06	0.30	0.55	1.00	0.13	0.04	-	0.01	0.29	7.2
0.6	0.07	0.39	0.67	0.71	0.15	0.06	0.04	0.02	0.31	7.2
0.7	0.07	0.37	0.66	0.70	0.16	0.07	0.13	0.02	0.32	7.3
0.8	0.07	0.34	0.63	0.68	0.17	0.08	0.27	0.03	0.31	7.3
0.9	0.07	0.31	0.58	0.65	0.17	0.09	0.44	0.04	0.30	7.4
1.0	0.06	0.28	0.52	0.52	0.16	0.10	0.63	0.05	0.28	7.4
1.1	0.06	0.26	0.46	0.52	0.15	0.10	0.80	0.06	0.25	7.3
1.2	0.06	0.19	0.34	0.38	0.12	0.09	1.19	0.07	0.22	7.3
1.5	0.04	0.14	0.25	0.27	0.10	0.08	1.47	0.08	0.19	7.2
1.75	0.03	0.10	0.17	0.18	0.08	0.06	1.79	0.09	0.16	7.1
2.0	0.02	0.07	0.11	0.12	0.05	0.04	2.00	0.10	0.13	6.9
2.5	0.01	0.03	0.05	0.05	0.02	0.02	2.32	0.10	0.08	6.6
3.0	-	0.01	0.02	0.02	0.01	0.01	2.49	0.10	0.05	6.4
4.0	-	-	-	-	-	-	2.19	0.05	0.01	5.9
5.0	-	-	-	-	-	-	2.12	0.03	-	5.6

Note. Density of the levels of the continuous spectrum is normalized to the density of neutron resonances. The cross sections everywhere smaller than 20 mb are omitted.

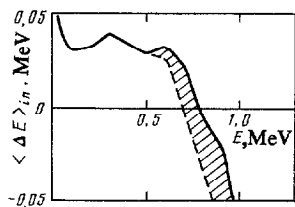


FIG. 3. The mean energy transferred to the neutron in the inelastic collision, $\langle \Delta E \rangle_{in} = \sum_i (\sigma_{m \rightarrow i} \Delta E_i) / \sigma_{tot}$. The solid curve: the density of levels above 1.64 MeV is normalized to the resonance density; the dashed curve: normalization to the number of discrete levels in the interval 1.14–1.64 MeV.

larger than 0.5 MeV is related to the fact that in this region levels of ^{180}Hf lying above 1.64 MeV are excited, the locations and quantum numbers of which are unknown. In order to take into account these discrete levels, one usually replaces them by a continuous distribution, the density of which is given most often in the form of semi-empirical Gilbert-Cameron formulas.¹⁹ Also other approaches have been proposed (in particular, microscopic ones) for the description of this density which take into account explicitly the influence of the deformation and the additional contribution from collective states (see Ref. 20). However, the improvement of accuracy in such calculations is accompanied by a significant increase of their laboriousness, which in our case hardly makes sense because of the significant uncertainty of other parameters. Therefore in the present paper the dependence of the levels on the excitation energy was calculated with the Gilbert-Cameron formula¹⁹ with the parameters of Cook *et al.*²¹ Here the resonance density 0.24 eV^{-1} calculated for ^{180}Hf in the vicinity of the neutron binding energy agrees with the experimental density¹⁷ $0.23 \pm 0.01 \text{ eV}^{-1}$. The number of levels of ^{180}Hf predicted in the interval of low excitation energies 1.14–1.64 MeV is here equal to 7 while the experimental value is 24. In order to estimate the influence of this discrepancy, the calculations of the cross sections for neutron scattering and radiative capture are performed with the density normalized to the number of neutron resonances but underestimating the number of low-energy levels, and also with the increased density corresponding to the number of low-energy levels (24) [normalization was performed by changing the parameter E_0 ; see Eq. (2) in Ref. 19]. The energy dependence of the radiative widths was estimated in the assumption of a dipole character of the emitted γ rays, with normalization to experimental values close to the neutron binding energy. Here the possible energy dependence of the radiation strength function (see Ref. 22) was not taken into account, because it can significantly influence the results only in the neutron energy range of the order of several MeV, where the role of the radiative channel is small.

Because of the large energy of the isomeric level, even at low energies of the incident neutron the $(n, \gamma n)$ reaction is possible, which decreases the cross section for radiative capture. In this reaction after the emission of the γ ray the excitation energy of the nucleus is still large enough for emission of a neutron. The cross sections for the reactions (n, γ) and

$(n, \gamma n)$ with the nucleus in the ground state are shown in Fig. 2, from which it is seen that for incident neutron energies $E \geq 2.5$ MeV the inclusion of $(n, \gamma n)$ decreases by several times the cross section for radiative capture.

In Table I we show cross sections for the inelastic acceleration, slowing down, and radiative capture calculated starting from the incident-neutron energy 1.1 keV. We note that the results cannot be extrapolated to the region of very low energies (for example, thermal), because there the influence of individual resonances begins to play a nonnegligible role.

The magnitude of the cross section for the acceleration is hundreds of mb and is larger than the cross section for the slowing down in discrete levels lying above the isomeric level. In Fig. 3 we show the mean energy $\langle \Delta E \rangle_{in}$ transferred to the neutron in one collision:

$$\langle \Delta E \rangle_{in} = \sum_i \frac{\sigma_{m \rightarrow i} \Delta E_i}{\sigma_t}, \quad (2)$$

where σ_t is the total cross section, and $\sigma_{m \rightarrow i}$ and ΔE_i are cross sections for excitation of the i th level and the change of the neutron energy related to it; here for acceleration ΔE_i is positive and for slowing down it is negative. For the upper estimate of the quantity $\langle \Delta E \rangle_{in}$ the calculations are performed with the density of levels of the continuous spectrum of ^{180m}Hf normalized to the density of neutron resonances, and for the lower estimate, normalized to the density of the discrete spectrum. As is seen from Fig. 3, up to the energy 0.7–0.8 MeV $\langle \Delta E \rangle_{in}$ is positive; thus the isomer ^{180}Hf is a "neutron accelerator" and not the usual moderator.

Thus, the isomer ^{180m}Hf permits the acceleration of slow neutrons up to energies of hundreds of keV.

The accuracy of the model calculations performed is not large because of the approximate character of the model used and uncertainties of the input parameters. The calculations are crude (for example, in addition to the approximations already mentioned the direct excitation of the rotational levels of the band constructed on the isomeric states, etc., was not taken into account). Because of this, it makes sense to study the isotope ^{180}Hf in detail: improvement of the level scheme, measurement of the strength functions and deformation and, of course, observation of the acceleration of not only thermal but also fast neutrons.

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