Non-Gaussian Uncertainty Distributions: Historical Trends and Forecasts of the United States Energy Sector, 1983-2010.

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Abstract

We apply a novel method of uncertainty parametrization and analysis to time-series data of recent supply and demand projections for the United States' energy sector. Based on determinations of the actual uncertainties in past forecasts (1983-1990) of over 170 energy producing and consuming sectors of the U. S. economy we develop a simple one-parameter model that can be used to estimate a probability distribution for future projections.

1 Introduction

Forecasting future energy consumption is a prerequisite for many major economic and policy decisions such as how best to reduce carbon dioxide emissions to alleviate global warming, or how best to stimulate the pace of development of alternate sources of energy. Sophisticated modeling systems are used to produce the most realistic possible projections. Model reliability is limited, however, in part because of the uncertainties inherent in any projection [1]. The range of uncertainty is usually estimated by running the model first under a set of assumptions deemed the most realistic (the *Base* or *Reference* case) and then under a few seemingly less probable but still reasonable assumptions. This procedure is commonly utilized to map out a confidence interval often summarized by *High* and *Low* estimates. The resulting ensemble of estimates, however, does not constitute a classical statistical sample, and can only be used to obtain a subjective characterization of the true probabilities.

The outputs of energy supply and demand models and forecasts are frequently used as input to decision theoretic models or are directly cited in policy analysis. Decision theory, however, requires that probability values be assigned to each alternative before risks and benefits can be compared [2]. A lack of formal statistical probability distributions for projections or extrapolations is encountered in a variety of disciplines, and various attempts have been made to surmount the resulting difficulties, including the elicitation of "subjective confidence intervals" [3] for model parameters.

It is well known, however, that there is a strong tendency for researchers to underestimate uncertainties in

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results, increasing the probability of "surprises" and decreasing the usefulness of the forecasts [3-5]. In this paper we build on earlier work [6-9] and develop a method to quantify the uncertainty in a time-series of historical forecasts for which the actual values are now known or can be estimated (see [10] for further details). The goal here is to apply revised uncertainty estimates to modify forecasts of future energy supply and demand to reflect the prior level of model accuracy.

The implicit assumption made in our work is that an estimate of the reliability of predictions can be derived from an examination of the way in which similar predictions made in the past actually turned out. Thus, this paper divides naturally into two parts: the characterization of uncertainty, particularly for low probability events; and application of our method to an existing set of forecasts.

2 Probability Distributions

Uncertainty in energy forecasts is usually presented in the form of "reference," "lower" and "upper" estimates (R, L, and U respectively) that are obtained by running a model with different sets of exogenous parameters (e.g. annual rate of growth or the size of a carbon emissions tax). Following [9,10] we assume that the range of parameter variation used by a forecaster represents a subjective judgment about the probability that the true value $T \in [L, U]$. We will use the convenient normalized measure of the deviation of the "old" (previously measured or projected) values, A, from the true value, a: $x = (a - A)/\Delta$. In this paper we will determine the actual distribution of x values from historical energy forecasts and show that it can be conveniently fit with exponential functions.

We apply this approach to the largest coherent set of US energy forecasts for the year 1990 A.D., the Annual Energy Outlook (AEO) published by the U. S. Department of Energy [11]. We then estimate the "credibility intervals" for future projections. The AEO is compiled using an integrated energy modeling system which includes supply modules for oil, coal, gas, and electricity markets, and a set of energy demand models. The supply models determine supply and price for each fuel conditional upon consumption levels, while the demand models determine consumption conditional upon end-use price. The forecasting module solves for market equilibrium for each fuel by balancing supply and demand to produce an energy balance for each forecast year [11].

The low, reference, and high (L, R, U, respectively) scenario forecasts are aggregated by fuel type within the supply module, and by end-use within the demand module. Over 170 separate supply and demand sectors are included in the model [11]. To assign a probability interval to (U - L) we follow the procedure described in [9] and construct a normal distribution with mean, (L+U)/2, (generally equal to the reference case R) and standard deviation, Δ, in such a way that the area between L and U is equal to a specified probability value, α. For α=95%, Δ=(U-L)/3.92, for α=68%, Δ=(U-L)/2.0, and for α=50%, Δ=(U-L)/1.35. We shall use α=68% in this paper and therefore calculate $x=2\cdot(T-R)/(U-L)$ where a is the actual value observed for the year in which L, R, U are forecast. This choice of α corresponds to the usual practice of splitting the difference between high and low estimates and using half this interval as a surrogate for the standard deviation. If the reference value, R, does not coincide with the mean value, (L+U)/2, then x depends on the sign of deviation T-R: x=(T-R)/(R-L) for $R > T$ and x=(T-R)/(U-R) if $R < T$ with $L < R < U$ assumed for both cases.

3 Results

We analyzed the AEO projections for 1990 made in 1983, 1985, and 1987 that consisted of 182, 185, and 177 energy producing or consuming sectors of the U.S. economy respectively. The variation in the number of sectors resulted because the low and high projections coincided in some cases, and no corresponding uncertainty range could be derived. In 47, 50, and 47 cases respectively, the x value exceeded 100; we conservatively assumed that this was not simply parametric uncertainty and omitted these cases. For all remaining cases the x values were calculated and the frequency distributions analyzed.

Figure 1(a) demonstrates that the distribution of signed

x values is approximately symmetric with respect to zero; there is no large systematic bias (e.g. a gross underestimation of energy consumption in all or many sectors). The correlation structure of the sectors between the 1983-1985, 1985-1987, and 1983-1987 AEO forecasts for 1990 are shown in Figures 1(b) - (d), respectively. The scattergrams are for signed x values less than 10. The largest linear correlation coefficient, $r = 0.55$, is observed between the 1983 and 1985 forecasts. The lack of consistent trends in the scattergrams of x values after the earliest model years is good evidence that the forecasts are generally independent.

Figure 2 shows the cumulative probability distributions of $|x|$ for the projections made for 1990 in 1983, 1985, and 1987 together with the Gaussian and exponential distributions. The three empirical distributions are strikingly similar. The similarity could be due in part to the modest correlation between the 1983 and 1985 forecasts (Figure 1(b)) although the lack of any such correlation between either of the later two forecasts (Figures 1(c) and (d)) suggests that this is not the case. Although the absolute error in forecasts made in 1987 for 1990 is somewhat smaller than made in 1983 for 1990, the range of uncertainty is also smaller so that probability of "large" deviations relative to the observed uncertainty is roughly the same as for the other two years. Initially we expected that energy forecasts for aggregated sectors of economy would be more reliable than projections for individual sectors. However, we found this not to be the case (Figure 2, heavy dashed line).

To illustrate how the exponential functional form might arise consider a set of estimates with the mean A and standard deviation Δ (see [9] for further details). Assume that the mean is unbiased but that the estimate of Δ is randomly biased by systematic errors with a distribution $f(t)$. The distribution for $x = (a - A)/\Delta$ is then no longer a simple Gaussian, but can be written instead as a compound distribution $p(x) \propto \int_0^\infty f(t) exp[-x^2/2t^2]dt/t$. It appears that if $f(t)$ is sufficiently broad so that at large t: $f(t) \propto exp(-t^2/2u^2)$ then we find that $p(x) \propto exp(-|x|/u)$. The new parameter, u, is the relative uncertainty in the original standard deviation, Δ.

The normal ($u = 0$) and exponential distributions ($u > 1$) are members of a single-parameter family of curves shown in Figure 3. In this framework the parametric uncertainty can be quantified by analyzing the record of prior projections and estimating the value of u. The cumulative probability functions for $u \geq 1$, $x \geq 3$, can be approximated with $e^{-|x|/(0.7 \cdot u + 0.6)}$. Our previous analysis shows that $u \sim 1$ for physical constants and $u \sim 3$ for current models of population growth [6-9]. Thus, while u is not necessarily the same for different types of forecasts, these data exhibit a consistent functional form that can be computed from a set of past projections and subsequent measurements of the true values.

4 Application to Existing Forecasts

Our method can be applied to current and future AEO energy projections by inflating the estimated uncertainty range with $u = 3$, corresponding roughly to an inflation by a factor of four (see the caption of Figure 4 for computational details). For example: in the current (1992) AEO the total U.S. production from nuclear power projected for the year 2010 is 6.9 Quads with U and L estimates set at 7.5 and 6.7 Quads, respectively [11]. We assume that this range corresponds to the 95% confidence interval of the forecaster.

Note that in estimating u values we assumed α=68% for the *old* forecasts but for the *current* projections we assume α=95%. In this way we account for the (hopefully) improved reliability of more recent forecasts. Had we assumed α=95% for the old forecasts, the derived standard deviations would be two times smaller and all x values would be two times larger. The resulting u values and the corresponding inflation factors would be also larger than the ones we used.

Based on u=3 in our compound exponential model we then forecast the 95% confidence interval to be from U = 9.4 to L = 6.2 Quads, as shown in Figure 4. This greatly decreased confidence suggests that without significant revision and recalibration it is prudent to apply the same skepticism to current and future AEO forecasts. These are shown in Figure 4 for three production sectors (crude oil, nuclear power and renewables) and three consumption sectors (liquified natural gas, coal, and residential electricity). The history of past projections suggests that the production from renewable sources (Figure 4(c)) in 2010 AD is expected to lie between 8 and 12.5 Quads, and may not fall within the parameter range 9.8 - 10.8 Quads of AEO analysis. Even this estimate is likely to be overconfident because of rapid developments in this environmentally benign production sector.

The revised projections for coal consumption (Figure 4(e)) are interesting in that the AEO forecasts already assume some negative environmental pressure on the coal industry. We find that in 1992, with no greenhouse gas regulations in effect, $x_{coal} = -2.91$. This suggests that the latest AEO model does not incorporate the uncertainty over whether industry can develop new coal technologies in the present uncertain atmosphere.

While domestic crude oil production has declined by almost 20% in the last decade, natural gas and in particular liquified natural gas (LNG) production and consumption (Figure 4(d)) increased sharply. Although the major causes are a decline in readily available oil reserves and an increase of known natural gas reserves respectively, the trend was in part driven by changes in demand and industry regulatory structure to the point that current production and delivery capacity is in excess of demand [11]. The AEO model did not anticipate the variability in the demand for LNG: we found that $x_{LNG} = 7.3$ for the period 1983 - 1990.

5 Discussion and Applications

An examination of past trends in measured values permits a characterization of the uncertainty and overconfidence in model parameters. Measurements from what is generally taken to be the "fundamental science" particle physics, provides a useful baseline case because physical constants do not change with time [12]. For a time-series set of measurements of elementary particle properties we previously [7,8] found $u \approx 1$. This value might be a reasonable lower limit on the uncertainty expected to appear in models involving substantial structural uncertainty.

Economic and environmental forecasts involve both parametric uncertainty in the models and evolution over time of the system. Direct "ground truth" measurements are available as we pass the target date of an old set of forecasts. For energy forecasts and projections of population growth [6,7,9] we find that the observed long tails are well fit by simple exponential functions with one additional parameter, u, which can be interpreted as the ratio of unsuspected systematic errors to the recognized uncertainties. Estimation of u for specific data sets provides a measure of the parametric confidence intervals that are applicable in scenario planning particularly when we are interested in probability estimates for events expected to lie far from the mean.

It is interesting to note that u values for three sets of projections of the U.S. energy consumption for 1990 made in 1983, 1985, and 1987 that encompass a range of different sectors all converge on $u \approx 3$. Furthermore, aggregating several sectors together, does not improve the situation. This suggests that although the absolute error in the 1987 to 1990 forecasts is smaller than in 1983 to 1990 forecasts, "degree of overconfidence", appears roughly the same. Note, that the purpose of this exercise is not to criticize the AEO; it is, in fact, a remarkably useful and sophisticated model. In fact we use the AEO model because a careful set of high, reference and low estimates are included, a practice that all forecasters should be required to emulate. Overconfidence is evidently endemic in model efforts. The goal here is to illustrate the problem and suggest methods to correct for this tendency.

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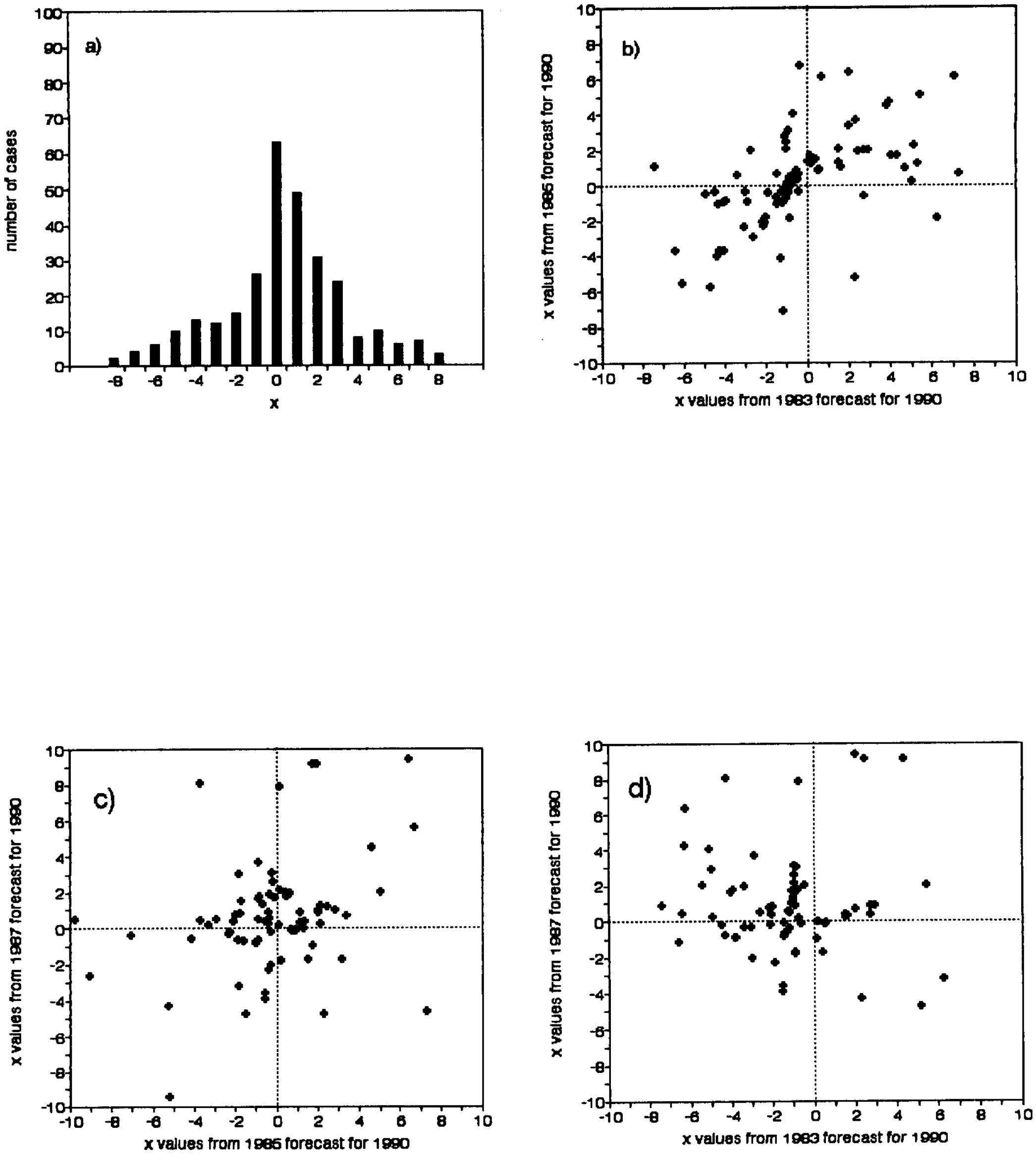

Figure 1: (a) Probability distribution of signed *x* values for Annual Energy Outlook projections. The data is an accumulation of the 1983, 1985 and 1987 values and is truncated at $|x| > 8$. (b) cross-correlation scattergram between the 1983 and 1985 (1983-1985) AEO forecasts for signed *x* values less than 10; (c) scattergram between the 1985-1987 AEO forecasts for 1990; (d) scattergram between the 1983-1987 AEO forecasts for 1990. The data demonstrate that there is no significant correlation between the 1983 and 1987, and 1985 and 1987 AEO forecasts. The two earliest AEO models, from 1983 and 1985, are moderately correlated (linear correlation coefficient, $r = 0.55$).

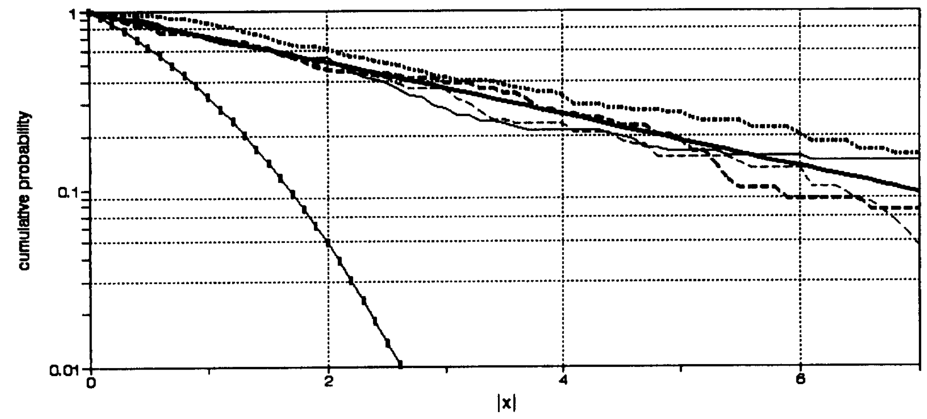

Figure 2: Annual Energy Outlook projections. The presentation is as in Figure 1: 1983 to 1990 (heavy dotted line); 1985 to 1990 (dashed line); 1987 to 1990 (solid line), totals (heavy dashed line); compound exponential distribution with u=3, $e^{-|x|/2.7}$ (0.7·3+0.6=2.7), (heavy solid line); Gaussian (thin solid line with vertical markers).

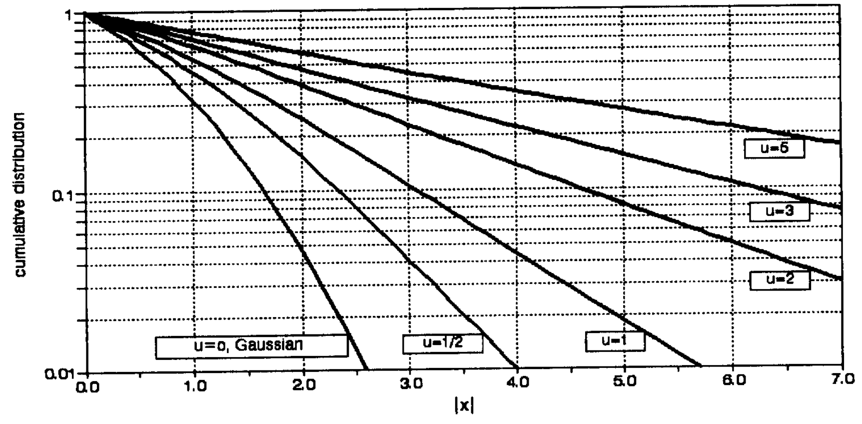

Figure 3: One-parameter set of probability distributions of deviations: parameter u defines the uncertainty in the standard deviation t of the Gaussian distribution. The values of u are indicated in the figure. The curves demonstrate the continuum of probability distributions: from Gaussian (u=0) to exponential (u > 1).

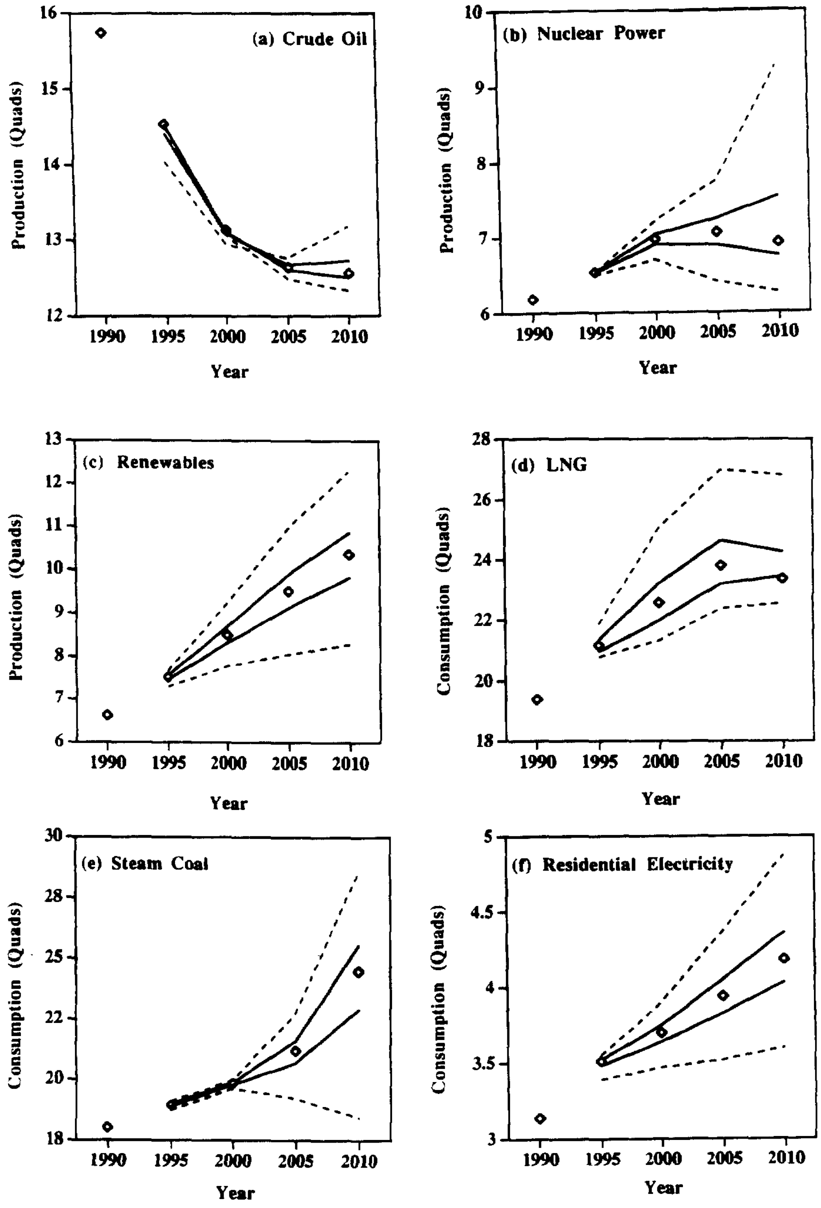

Figure 4: Confidence intervals for six production and consumption sectors in the AEO database. The diamonds are for the Reference Case, with the 1990 value that actually reported by the Energy Information Administration [11]; the *Low* and *High* confidence limits as reported in the AEO are shown as solid lines; our re-calculation of the confidence limits using $u = 3$ throughout are shown as dashed lines. In (a) the production estimates for crude oil, including other hydrocarbons from drilling, is shown; Nuclear power generation (b); (c) renewables (including both utility and non-utility generating capacity) are projected at 2.3%. Consumption projections for several sectors are also shown: (d) liquid natural gas; (e) "steam coal"; and (f) residential electricity demand.

References

[1] K. C. Land and S. H. Schneider, "Forecasting in the Social and Natural Sciences: an Overview and Analysis of Isomorphisms", in *Forecasting in the Social and Natural Sciences*, K.C. Land and S.H. Schneider, eds., (D. Reidel Publishing Company: Dordrecht, Holland, 1987).

[2] H. Raiffa, Decision Analysis: *Introductory Lectures on Choice Under Uncertainty* (Addison-Wesley: Reading MA, 1968).

[3] M. G. Morgan and M. Henrion, *Uncertainty: A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis*, Cambridge University Press, New York, 1990.

[4] V. Zarnowitz, *Business Cycles: Theory, History, Indicators, and Forecasting*, University of Chicago Press, 1992.

[5] W. D. Nordhaus and G. W. Yohe "Future Carbon Dioxide Emissions from Fossil Fuels", in *Changing Climate: report of the Carbon Dioxide Assessment Committee,* National Academy Press, Washington, D.C., p. 87 - 150, 1983.

[6] A.I. Shlyakhter and D.M. Kammen "Sea-level rise or fall?" *Nature*, 357, 25, 1992.

[7] A.I. Shlyakhter and D.M. Kammen "Estimating the range of uncertainty in future development from trends in physical constants and predictions of global change," CSIA discussion paper 92-06, Kennedy School of Government, Harvard University, July 1992.

[8] Shlyakhter A.I., Shlyakhter I.A., C.L. Broido, and R. Wilson "Estimating uncertainty in physical measurements, observational and environmental studies: lessons from trends in nuclear data," these proceedings.

[9] A.I. Shlyakhter and D.M. Kammen "Estimating the range of uncertainty in future development from trends in physical constants and population projections," these proceedings.

[10] A.I. Shlyakhter, D.M. Kammen, C.L. Broido and R. Wilson "Quantifying the Credibility of Energy Projections from Trends in Past Data: the U.S. Energy Sector," paper submitted to *Energy Policy.*

[11] *Annual Energy Outlook with projections to 2010*, Energy Information Administration, Office of Integrated Analysis and Forecasting, US Department of Energy, Washington, DC 20585, DOE/EIA-0383(92), 1992.

[12] A.I. Shlyakhter "Direct test of the constancy of the fundamental nuclear constants," *Nature*, 264, 340, 1976.