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## Estimating the Range of Uncertainty in Future Development from Trends in Physical Constants and Predictions of Global Change

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Estimating the Range of Uncertainty in Future Development
from Trends in Physical Constants and Predictions of Global Change.

Alexander :. Shlyakhter and Daniel M. Kammen

Science policy frequently hinges on reliable assessment of the uncertainty in predictions derived from various models. Because of systematic and random errors in the parameters of scientific models, measured values are commonly considered to be stochastic quantities following a Gaussian distribution, with the uncertainty characterized by the estimated standard deviation. We present analysis of multi-decade trends in the reported values of fundamental physical constants and projections of future energy demand and population growth, which reveals that a Gaussian, or normal, distribution grossly underestimates the frequency of events lying very far from the mean. The probability of large deviations is instead well fit by a simple exponential. This asymptotic behavior appears naturally in a compound distribution where both the mean and standard deviation are normally distributed stochastic quantities. We illustrate this formulation by estimating the probability of catastrophic sea-level rise resulting from global warming.

**Estimating the Range of Uncertainty in Future Development from Trends in Physical Constants and Predictions of Global Change.**

Alexander I. Shlyakhter and Daniel M. Kammen

It is well known that researchers tend to underestimate the systematic uncertainties in their results[1,2]. This bias increases the probability of "surprise" and is highlighted by the question: If calculations of sea-level rise by 2050 A.D. predict 33 ± 32 cm what is the probability of extreme rise of 150 cm? In this paper we outline a new approach to uncertainty characterization that utilizes data on the history of previous projections in order to quantify the effects of systematic uncertainty in complex models[3].

Uncertainty analysis is based on the standard sampling theory which assumes that individual measurements are drawn from the same population and are randomly distributed around the true value of the measured quantity. A set of measurements forms a sample of the general population. By the Central Limit Theorem under very general conditions the distribution of a large sample is asymptotically Gaussian. The presence of systematic uncertainties, violates the assumption of unbiased sampling. In practice, however, the Gaussian distribution is still implicit when reporting the measured values and their uncertainties[4]. For large deviations from the estimated sample means the Gaussian distribution is negligibly small and can underestimate the true probability by many orders

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of magnitude. In this paper we describe a practical approach which utilizes trends in previous measurements and model predictions to quantify the range of uncertainty in current models.

Bukhvostov[5] compared the measured values of 235 elementary particle properties (mass, nuclear moment and lifetime) as compiled in 1964[6] to a more recent compilation from 1972[7]. For 181 of the measurements, the reported standard deviations were smaller by at least a factor of four: these new values were taken to be "exact." A convenient measure of the deviation of "new" (1972) values from the "old" (1964) values is $x = (a - A)/\Delta$, with a the exact value, A the measured value, and $\Delta$ the old standard deviation. The resulting experimental distribution of $|x|$ has a tail extending beyond five standard deviations[5] (Figure 1; dotted line). By contrast the Gaussian distribution

$$p(x) \quad \frac{1}{\sqrt{2\pi}} e^{\frac{x^2}{2}}$$

predicts that probability of such a fluctuation is only $5.6\,10^{-7}$. As presented in Figure 1, a simple exponential distribution, $e^{-|x|}$, describes the data remarkably well.

To illustrate how this distribution arises recall that if only normally distributed (Gaussian) fluctuations were present, the standard deviation could be calculated with high accuracy. Systematic uncertainties are more difficult to incorporate. Therefore it is only natural to consider not only the measured value of the mean but also the estimated standard deviation of that mean as stochastic quantities. Following Bukhvostov[5] we assume that both the measured value A and its calculated standard deviation $\Delta'$ are distributed around their true values a and $\Delta$ respectively; we denote this distribution by f(t) where $t=\Delta'/\Delta$. Assuming the normal distribution from Eq.(1) for each value of $\Delta'$, we obtain:

$$p(x) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{dt}{t} f(t)\, e^{\frac{x^2}{2t^2}}$$

If f(t) has a sharp peak near $t=1$, Eq. (2) reduces to the normal distribution Eq.(1). If f(t) is broad, however, the result is different. For simplicity, we shall consider only the asymptotic behavior of p(x) when $|x| \gg 1$. In this case, the value of the integral in Eq.(2) is determined by the asymptotic behavior of f(t) as $t \to \infty$, since for small t the exponent is nearly zero. Let us assume that for $t >> u$ f(t) follows a Gaussian law:

$$f(t) \sim e^{\frac{t^2}{2u^2}}$$

where $u=\delta/\Delta$. The new parameter u, comprises the unknown systematic component of the total error and quantifies the uncertainty $\delta$ in the estimation of $\Delta$. At $x \gg 1$ the main contribution to the integral in Eq.(2) comes from the vicinity of the saddle-point where the exponential term in Eq. (2) reaches a maximum at $t=t_{max}$: $t^2_{max}=u|x|$. For $x \gg 1$, the probability distribution is not Gaussian but exponential:

$$p(x) \sim e^{-\frac{|x|}{u}}$$

The preceding analysis not only provides a natural explanation of the surprisingly frequent occurrence of extreme values in many models, but also suggests that Gaussian and exponential distributions might be related in a straightforward fashion by a single parameter u (Figure 2). If we assume that f(t) is a Gaussian distribution with the mean value t=1 and standard deviation u (truncated at t=0 to exclude negative values of t and renormalized accordingly) then integrating Eq. (2) gives the cumulative probability S(x) of deviations exceeding $|x|$:

$$S(x)=\frac{\sqrt{2}}{\sqrt{\pi}\,u\cdot \mathrm{erfc}\left(-\frac{1}{\sqrt{2}\,u}\right)}\int_0^{\infty} e^{-\frac{(t-1)^2}{2u^2}}\,\mathrm{erfc}\left(\frac{|x|}{t\sqrt{2}}\right)dt$$

For u=0 Eq. (5) is reduced to $S(x)=\mathrm{erfc}(|x|/\sqrt{2})$. On the logarithmic scale this curve is parabolic while the exponential curve is linear. The exponential term apparently dominates the distribution at $x \geq 3$ even at $u \leq 0.25$. Ignoring this term at $u \sim 1$ would lead to underestimation of the probability of extreme events ($x \geq 5$) by many orders of magnitude. The exact values of S(x) depend on the probability distribution function for systematic uncertainties which is unknown. Here we used the Gaussian form because of the asymptote at $t \gg 1$ (Eq. (3)) which yields simple exponential behavior for $|x| \gg 1$; however other forms (e.g. gamma or lognormal distribution) could also fit the data and the choice of the most appropriate probability distribution for systematic uncertainties deserves further investigation.

In the above discussion random errors in the standard deviation that are due to finite sample size were not explicitly included. Given the sample size, N, the magnitude of the uncertainty due to these random errors can be calculated with the Student distribution for x with N - 1 degrees of freedom. For large N (N > 30) the Student distribution is asymptotically Gaussian while for N small the tails are considerably longer[10]. Reported values of physical constants typically are based on hundreds or thousands of individual measurements; in the case of Bukhvostov's data, for example, errors due to finite sample size are negligible. In Figure 2 the Student t-distribution for N = 11 (d.f. = 10 degrees of freedom) is plotted (thin solid line), demonstrating the upper limit on the uncertainty that can be explained purely in terms of random fluctuations.

The long record of measurements of elementary particle masses, lifetimes, and branching (decay) ratios has prompted several studies of the temporal evolution of systematic errors. Henrion and Fischoff[2] used the percentile of events that fall outside an assessed 98% confidence interval (i.e. $x > 2.33\ \Delta$) as the "surprise index" (SI). Overconfidence exists if significantly more than 2% of the measured events lie outside the predicted interval. Henrion and Fischoff[2] analyzed 306 measurements of elementary particle characteristics (92 kaon and hyperon lifetimes and 214 masses of associated resonance states) which were compiled in 1975 and compared with the "exact" 1982 values. The value of the surprise index was 7%. Analysis of particle properties through 1990 including prior results[2,8] is plotted in Figure 1. Remarkably all four data sets are well described by a simple exponential probability distribution, $e^{-|x|}$ (thick solid line) while the normal distribution, $\mathrm{erfc}(|x|/\sqrt{2})$, (thin solid line) describes the data only for $x < 0.5$.

Models used to forecast future trends in resource use and population growth may also exhibit over- or under-confidence on the part of investigators. In Figure 3 forecasts of the primary energy demand for the United States (thick dotted line) for the year 2000 A.D., and the population projections for 1985 A.D. made in 1973 for 133 nations (thin dotted line), are analyzed in terms of the SI. Both exhibit significant systematic uncertainty, as described below.

The projections of U. S. energy requirements range from a high of almost 200 Quads (1 Quad $= 1.05 \times 10^{18}$ joules) published in 1972 to estimates as low as 50 Quads today[9], and come from a range of sources (environmentalists, power industry officials, and government agencies). The systematic uncertainties in the 69 estimates vary widely and reflect a diverse set of biases and model assumptions. The uncertainty due to random errors resulting from finite sample size can be easily estimated. The 69 estimates fall into 6 chronological bins with at least 11 estimates per bin; this corresponds to 10 degrees of freedom. The corresponding cumulative probability distribution expected according to the Student distribution (d.f. = 10) is plotted in Figure 2, and represents limits that might be considered statistically insignificant. The considerable discrepancy between the cumulative probability derived from the energy projections (Figure 3, heavy dashed line) and from the Student distribution indicates the dominance of systematic uncertainty in the overall error.

The data base of population projections provides an opportunity to both test our the method by calculating the cumulative probability distribution for past estimates and comparing the result to existing census figures, and to refine projections of future population growth. To address the former we analyzed United Nations population projections for the year 1985 (the most recent year with a completed census taken as the "exact" values) made in 1973[11]. The data base consists of 164 individual estimates in the form of "high" and "medium" and "low" variants for each nation. For a number of countries, census error, international migration, or politically motivated reporting inaccuracies introduced huge uncertainty, ranging up to $|x| > 300$. We limited the data base to $|x| < 10$ (consisting of N = 133 nations) and plotted the distribution in Figure 3 (dotted line). Because all the population estimates come from the same source, namely the United Nations it might be expected that systematic errors would be small, representing a well-calibrated set of projections. The uncertainty, however is very large, characterized by $u \sim 3$. We next plan to analyze the data on a geographic basis and extend the results to population projections for the year 2000 and beyond.

We chose to look at energy and population projections because of their importance for assessments of global climate change. Estimating changes in global sea-level due to greenhouse warming is a natural application of this technique of uncertainty characterization. Oerlemans, for example, projects sea-level rises with errors comparable to the estimates themselves: 33 ± 32 cm in 2050 and 65 ± 57 cm in 2100[12]. For sea-level rise there is no long history of projections that we can use to estimate the value of the parameter u. Error estimates in this case are little more than educated guesses; for simplicity we shall assume $u=1$ (which still may underestimate the probability of surprise). Extreme sea-level rise, of perhaps 150 cm in 50 years, is of prime regulatory concern and is evidently poorly characterized by a Gaussian probability distribution.

A comparison of Gaussian and exponential threshold probabilities for sea-level rise by 2050 and 2100 A.D. is presented in Figure 4. In the "business-as-usual" scenario[12,13] the normal distribution places the probability in 2050 A.D. of extreme sea-level rise greater than 1 meter at 0.5% in contrast to the 5% probability based on an exponential distribution: a difference of an order of magnitude.

The approach presented here uses empirical data on the distribution of errors in prior measurements to quantify the probability of errors in posterior measurements. In both the method presented here and in the Bayesian approach additional data obtained over time are used to refine the initial (prior) distribution of values for an uncertain parameter[17]. However, our method is, in fact, a classical statistical approach[8]: we quantify the deviations from the point parameter values, as opposed to iteratively refining posterior distributions. We have demonstrated that if the uncertainty in the standard deviation is large then the far tails of probability distributions are well characterized by exponential functions. In some cases the long tails of probability distributions resulting from simplified models can be truncated on the basis of additional constraints external to the model itself. For example, a simple statistical model for absorption of thermal neutrons by atomic nuclei predicts that there is a 5% chance to find an absorber 100 times stronger than expected and 1% chance that it will be 3000 times stronger. However, this model neglects the small widths of neutron resonances, $\Gamma$, compared to the average spacing, D; for very strong absorbers this assumption is incorrect and the distribution can be truncated at $(D/\Gamma)^2$ [18,19]. The search for such constraints is critical to the calibration and validation of realistic models.

Fundamental physical constants appearing in the laws of nature are generally considered to be the most reliably known parameters, yet analysis of their trends indicates widespread overconfidence in the completeness of our knowledge. We expect that similar effects should be even more striking in other fields. The approach taken here, that of estimating the parameter u, provides a novel means to quantify the uncertainties in scientific models. Our findings suggest that the parametric uncertainty of current models could be quantified by analyzing the record of prior projections and estimating the value of u. From the Figures 1 and 3 we see that u ~ 1 for physical constants and projections of energy demand, and u ~ 3 for current models of population growth. It is the goal of this Report to encourage other researchers to quantify the predictive capabilities of their models by utilizing the historical trends in parameter values from previous studies.

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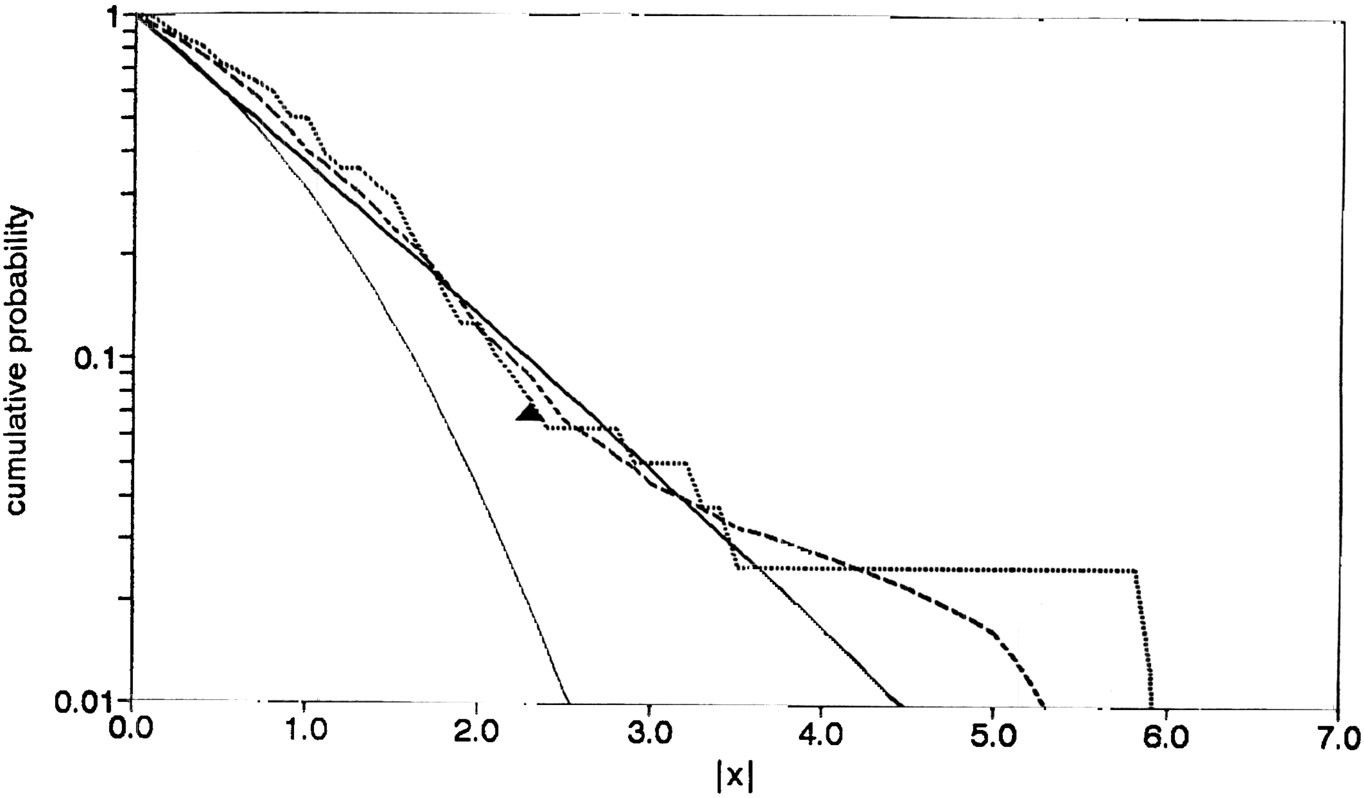
## FIGURE CAPTIONS

FIGURE 1. Probability of unexpected events in elementary particle data. The plots depict the cumulative probability, $S(x)=\int_x^\infty p(t)dt$, that new measurements (a) will be at least $|x|$ standard deviations ($\Delta$) away from the old results (A); $|x| = (a - A)/\Delta$ as defined in the text. Bukhvostov's[5] survey (1972) of particles stable in strong interactions for which the standard deviation was at least four times less than the value estimated in 1964 is shown as the heavy dashed line. A distribution updated to include particle data through 1990, as maintained by the Lawrence Berkeley Laboratory Particle Data Group[6], is plotted for the same selection criteria ($r=\Delta_{old}/\Delta_{new}=4.0$) as the heavy dotted line. The value of SI = 16%. The single triangle denotes the SI calculated by Henrion and Fischoff (H-F)[2,8] from the data set described in the text. Also plotted is a normal distribution, $\mathrm{erfc}(x/\sqrt{2})$, (thin solid line), and a simple exponential probability distribution, $e^{-|x|}$ (heavy solid line), which provides a good fit to the data.

FIGURE 2. One-parameter set of probability distributions of deviations: parameter u defines the uncertainty in the standard deviation t of the Gaussian distribution in Eq. (2). The values of u are indicated in the figure. The curves demonstrate the continuum of probability distributions: from Gaussian ($u=0$) to exponential ($u > 1$). In accordance with Eq. (5), the curve $u=1$ has the same asymptotic slope as $e^{-|x|}$ although they differ by a factor of about 2 that stems from the deviation from exponential behavior at small $|x|$.

FIGURE 3. Energy and population projections The presentation is as in Figure 1: energy (heavy dotted line); population (heavy dotted line); exponential distribution $e^{-|x|}$ (heavy solid line); Gaussian (thin solid line). The 69 projections of US energy demand for the year 2000 A.D. are from a variety of sources[9,14] and were made between 1972 and 1989. Estimates published as a range, for example 80 - 100 Quads, are taken as two separate values representing the endpoints. The data is grouped into six bins, each containing at least 11 estimates, and a mean and standard deviation are calculated for each bin. The "true" value for the year 2000 A.D. is taken to b3e an average over the most recent (1990) set of 8 estimates. Also plotted (heavy broken line) is the Student distribution with 10 degrees of freedom; d.f. = 10. The population data base includes projections from 164 nations with population exceeding 100,000and consists of an average of the "high"and "low" variants of the United Nations Population Studies series[11,15,16]. The projections were made in 1972 for the year 1985, which is the most recent year that the United Nations has released documented population figures for all of the countries in the data base. Data for 31 countries was excluded due to extreme errors (up to $|x| = 300$) that resulted from unanticipated international migration (frequently war refugees between relatively small nations), reliability questions surrounding particular census efforts, and clear cases of politically motivated reporting bias. Data for 133 nations satisfying the criteria $|x| < 10$ are included in the study. If one assumes that high and low estimates encompass 50% confidence interval the uncertainty in the data corresponds to $u \sim 3$.

FIGURE 4. Projections of sea-level rise for 2050 A.D. and 2100 A.D. The probability of a sea-level rise greater than a given threshold are plotted for the normal probability (2050: thin solid line; 2100: thin dashed line) and for the exponential distribution $e^{-|x|}$ (2050: heavy solid line; 2100: heavy dashed line). The data points are the projections based on the model of Oerlemans[12] with a simple fit to temperature perturbation based on a "Business-as-Usual" emission of greenhouse gases: $T = \alpha(t - 1850)^4$, where t is time, $\alpha = 27 \times 10^{-8}$ °K $yr^{-3}$ and $\Delta$ is 35% of the mean. The uncertainty in individual contributions to changes in sea-level are characterized by independent normal probability distributions, hence: $\Delta^2 = \Delta^2_{glac} + \Delta^2_{ant} + \Delta^2_{green} + \Delta^2_{wais} + \Delta^2_{expa}$ + internal variability, where the subscripts refer to the effect of glaciers, the Antarctic, Greenland and West Antarctic ice sheets and thermal expansion of water. Our fit to Oerlemans' calculation for 2050 A.D. results in sea-level rise of 33 ± 26 cm, and for 2100 66 ± 48 cm. The uncertainty in parameter values does not preclude a fall in sea-level (a negative sea-level rise).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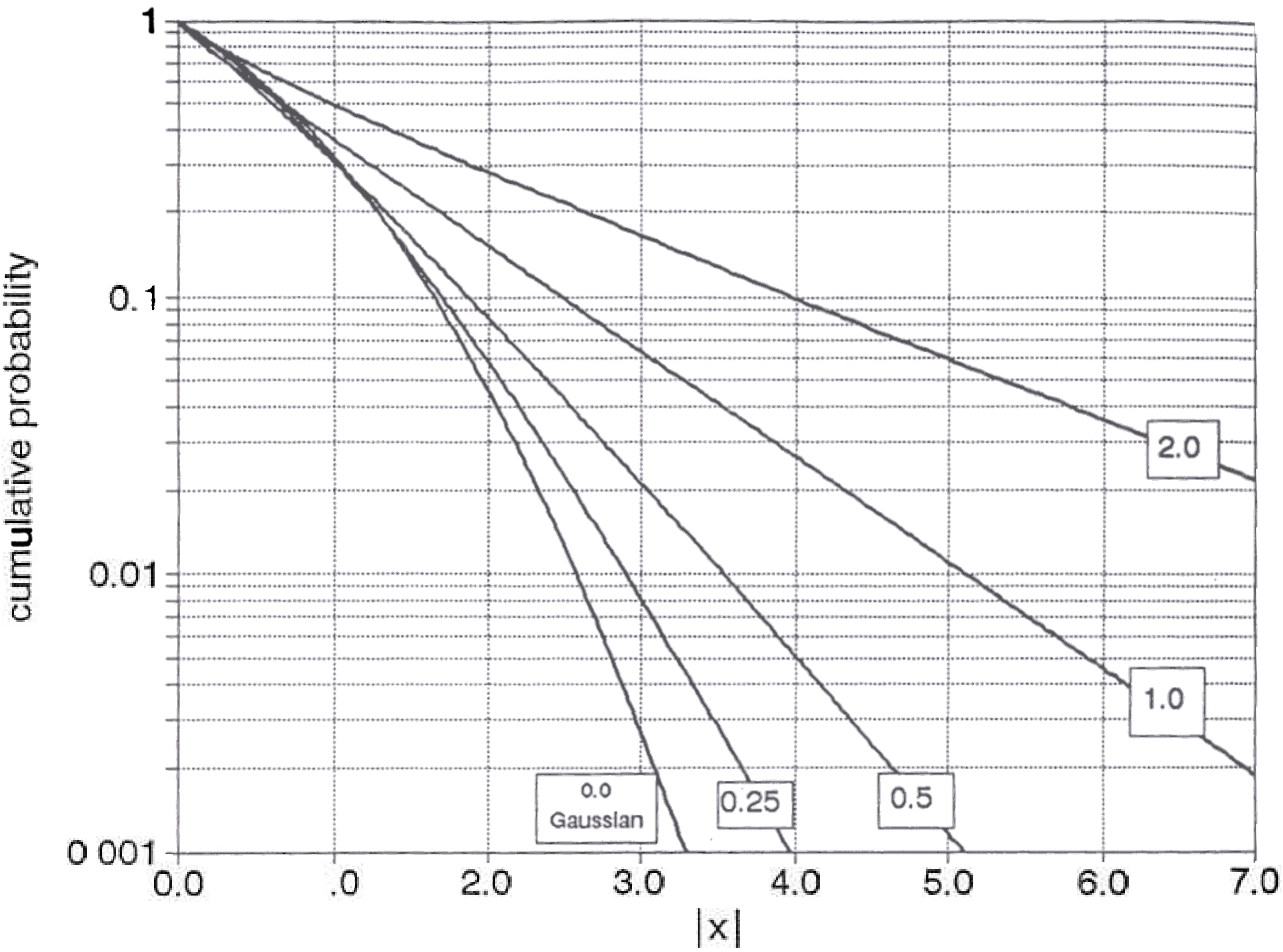
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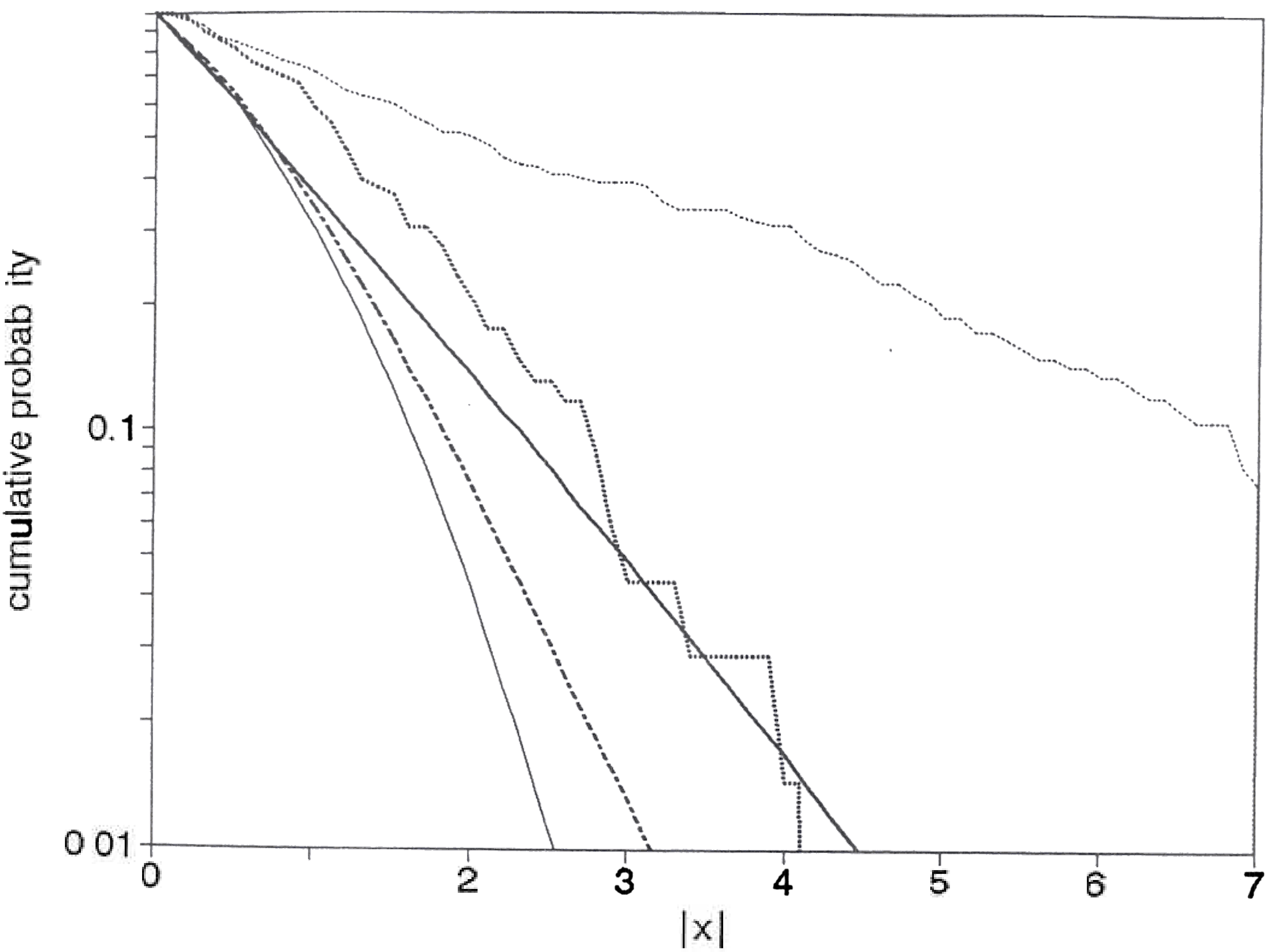

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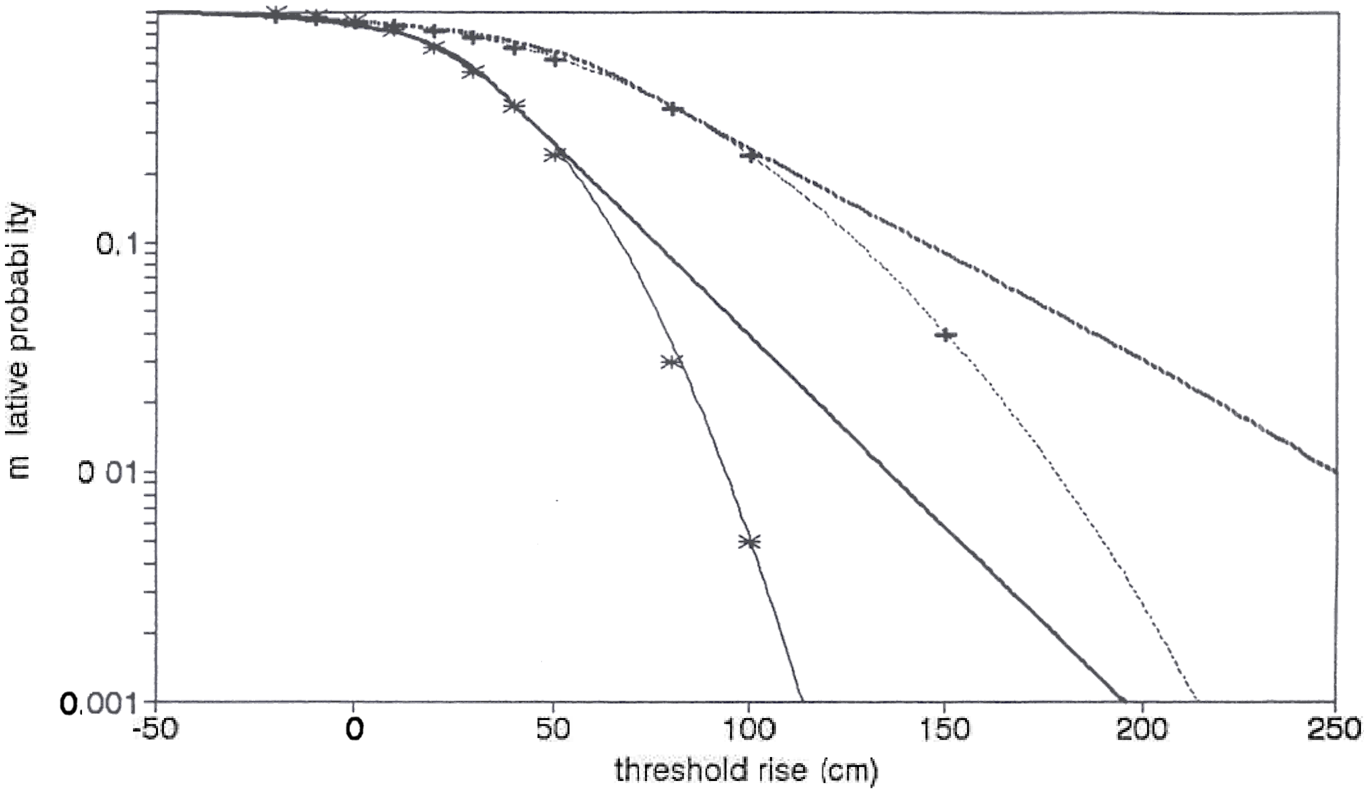

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